



Exercise 1. Show that $u(x, y) = x^3 - 3xy^2$ and $u(x, y) = e^{x^2-y^2} \cos 2xy$ are solutions of the two-dimensional Laplace equation $\Delta u = 0$.

Exercise 2. Show that

$$u(x, t) = \frac{1}{\sqrt{4\pi c^2 t}} \exp\left(-\frac{x^2}{4c^2 t}\right)$$

is a solution of the one-dimensional heat equation $u_t = c^2 u_{xx}$.

Exercise 3. Show that $u(r, \theta) = \ln r$ and $u(r, \theta) = r \cos \theta$ are both solutions to the PDE

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0.$$

Exercise 4. Let f be a differentiable function of one variable. Show that

$$u(x, t) = f(x - vt)$$

is a solution to the one-dimensional transport equation

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial x} = 0.$$

Use this fact to find the solution that satisfies the *initial condition*

$$u(x, 0) = \frac{1}{x^2 + 1}.$$

[*Suggestion:* Set $t = 0$ above and solve for f .]