Partial Differential Equations — Assignment 12

Spring 2018 — Due March 6

Exercise 1. Textbook exercise 3.6.3. [*Suggestion:* Use a scaled version of the series in Exercise 2.2.5.]

Exercise 2. Textbook exercise 3.6.4. [*Suggestion:* Use Exercise 2.3.6.]

Exercise 3. Show that the solution to the constant boundary flux heat problem

$$\begin{aligned} &u_t = c^2 u_{xx}, && 0 < x < L,\ 0 < t,\\ &u_x(0,t) = -F_1, \quad u_x(L,t) = -F_2, && 0 < t,\\ &u(x,0) = f(x), && 0 < x < L, \end{aligned}$$

is given by

$$\begin{aligned} u(x,t) =& \frac{F_1 - F_2}{2L} x^2 - F_1 x + \frac{c^2(F_1 - F_2)}{L} t + \frac{(F_2 + 2F_1)L}{6} + \widehat{a_0}\\ &+ \sum_{n=1}^{\infty} \left(\widehat{a_n} + \frac{2L\left((-1)^n F_2 - F_1\right)}{n^2\pi^2} \right) e^{-\lambda_n^2 t} \cos\left(\frac{n\pi x}{L}\right), \end{aligned}$$

where $\lambda_n = \frac{cn\pi}{L}$ and

$$\widehat{a_0} + \sum_{n=1}^{\infty} \widehat{a_n} \cos\left(\frac{n\pi x}{L}\right)$$

is the $2L$-periodic cosine expansion of $f(x)$.

Exercise 4. Solve the constant boundary heat flux problem

$$\begin{aligned} &u_t = 2u_{xx}, && 0 < x < 3,\ 0 < t,\\ &u_x(0,t) = -1, \quad u_x(3,t) = 6, && 0 < t,\\ &u(x,0) = \begin{cases} 2 - x & \text{if } 0 < x \le 2,\\ 0 & \text{if } 2 < x < 3. \end{cases} \end{aligned}$$

[*Suggestion:* Use the previous exercise and Exercise 2.3.8.]