PARTIAL DIFFERENTIAL EQUATIONS
SPRING 2018

ASSIGNMENT 13
DUE MARCH 8

Exercise 1. Consider the radiating end heat problem

$$\begin{aligned}
&u_t = 9u_{xx}, \;\; 0 < x < 2, \;\; t > 0, \\
&u(0,t) = 0, \;\; t > 0, \\
&u_x(2,t) = -4u(2,t), \;\; t > 0, \\
&u(x,0) = \begin{cases} 100 & \text{if } 0 < x \le 1, \\ 0 & \text{if } 1 < x < 2. \end{cases}
\end{aligned} \tag{1}$$

a. Solve (1) using a generalized Fourier series as we did in class. Express your answer in terms of the positive solutions μ_n to $\tan(\mu L) = -\mu/\kappa$.

b. For $1 \le n \le 5$, numerically evaluate μ_n and the generalized Fourier coefficients c_n to four decimal places.

c. Use part **b** to write out the first 5 terms of the solution to (1).

Exercise 2. Textbook exercise 3.6.10.

Exercise 3. Show that if $\mu > 0$ satisfies $\tan(\mu L) = -\mu/\kappa$ ($\kappa, L > 0$), then

$$\int_0^L \sin^2(\mu x)\, dx = \frac{\kappa L + \cos^2(\mu L)}{2\kappa}.$$

[*Suggestion:* Use a half-angle formula to integrate sine squared.]