

Partial Differential Equations

Exercise 1. Show that the following functions are harmonic, i.e. solve the Laplace equation $\Delta u=0$. Feel free to use either Cartesian or polar coordinates, whichever seem more appropriate.
a. $u(x, y)=\frac{x^{2}-y^{2}}{\left(x^{2}+y^{2}\right)^{2}}$
b. $u(x, y)=x \log \sqrt{x^{2}+y^{2}}-y \arctan \left(\frac{y}{x}\right)$
c. $u(x, y)=x^{3}-3 x y^{2}$

