Partial Differential Equations
Spring 2018

Assignment 20
Due April 17

Exercise 1. Textbook exercise A.5.7 (find explicit expressions for the coefficients of two linearly independent solutions).

Exercise 2. Show that the change of variable $t = \sqrt{2x}$ in the ODE

$$2xy'' + y' + y = 0, \quad x > 0, \tag{1}$$

results in the constant coefficient ODE

$$\ddot{y} + y = 0,$$

where dots indicate derivatives with respect to t. Use this to find the general solution to (1).

Exercise 3. One can show that the ODE

$$4x^2y'' - 14xy' + (20 - x)y = 0 \tag{2}$$

has solutions of the form

$$y = x^r \sum_{n=0}^{\infty} a_n x^n, \quad a_0 \neq 0 \tag{3}$$

in which the series converges for all x.

a. By substituting (3) into (2), determine the values of r that yield solutions, as well as the recursion relations satisfied by the coefficients for each such value.

b. For each value of r found in part **a**, find the first 5 terms in the solution (3) (assuming $a_0 = 1$).