# Math 3357 Spring 2017

## Partial Differential Equations

### First Exam

### Due Tuesday, February 28

Your name (please print):

**Instructions:** This is a closed book, closed notes exam. Once you begin the exam you will have **three (3) consecutive hours** to complete it. Although you may feel free to use a calculator, **the use of other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use.

**The Honor Code requires that you neither give nor receive any aid on this exam.**

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

**Pledged:** ______________________________

Do not write below this line

| Problem | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| Points | 10 | 16 | 16 | 14 | 10 | 16 | 18 |
| Score | | | | | | | |

**Total:**____________________

**1.** Verify that $u(x,t) = \dfrac{x}{t+1}$ is a solution to $u_t + uu_x = 0$

**2.** Find the solution to the partial differential equation

$$x\frac{\partial u}{\partial x} = u^2 - (x+y)\frac{\partial u}{\partial y}.$$

that satisfies $u(x, 0) = f(x)$.

**3.** Use a linear change of variables to solve the partial differential equation

$$u_x - 2u_y = u^3 + \frac{1}{u}.$$

**4.** Complete the following statement of the Fourier convergence theorem.

**Theorem 1.** *Suppose that $f$ is a $2p$-periodic* ______________________________ *function. The Fourier series of $f$ is given by*

$$a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\left(\boxed{\quad}\right) + b_n \sin\left(\boxed{\quad}\right)\right)$$

*where*

$$a_0 = \boxed{\qquad\qquad}$$

*and*

$$a_n = \boxed{\qquad\qquad\qquad} \qquad\qquad b_n = \boxed{\qquad\qquad\qquad}$$

*for $n \geq 1$. The Fourier series converges to*

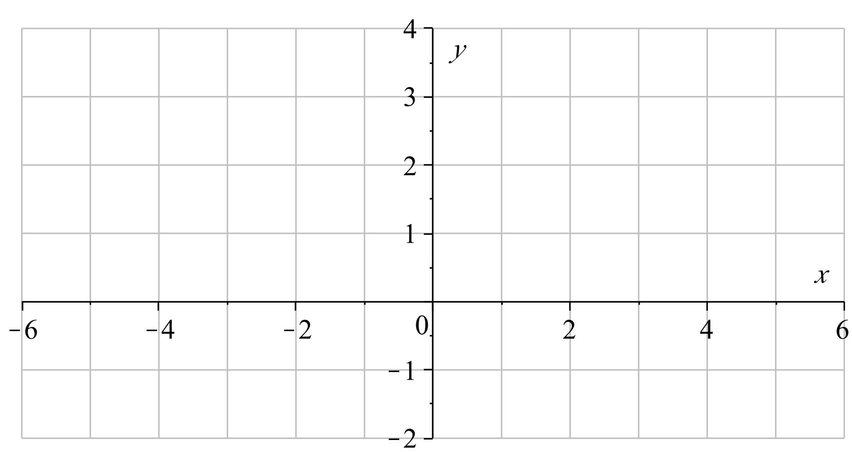
$$\boxed{\qquad\qquad}$$

*for all $x$.*

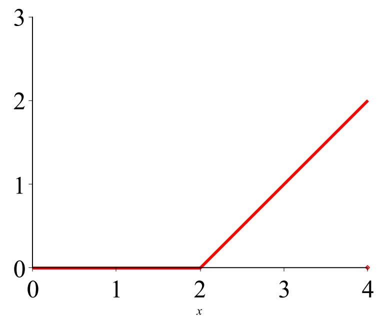
**5.** Let

$$f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1/2 \text{ or } 1/2 < x < 1, \\ -1 & \text{if } x = 1/2, \\ 3|x-2| & \text{if } 1 \leq x < 3, \\ f(x+3) & \text{otherwise.} \end{cases}$$

Carefully sketch the graph of the Fourier series for $f(x)$ on the interval $-6 \leq x \leq 6$.



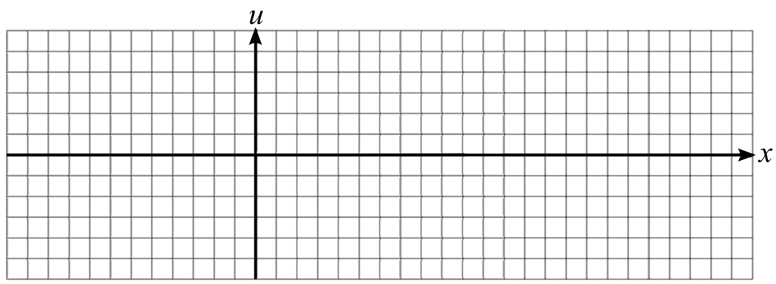
**6.** Let $f$ denote the function whose graph is shown below.



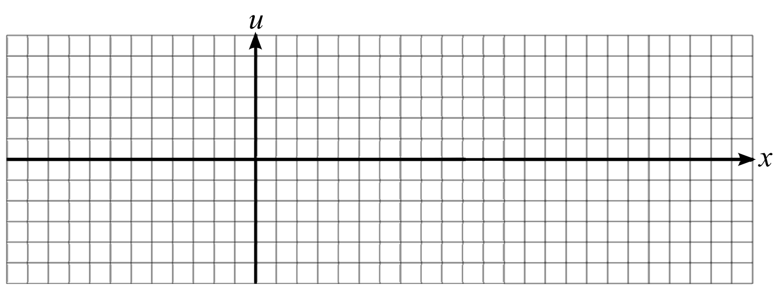

$$y = f(x)$$

**a.** Carefully sketch the graphs of the 8-periodic even and odd extensions $f_e$ and $f_o$ of $f$. Draw at least 2 periods of each.

$f_e$

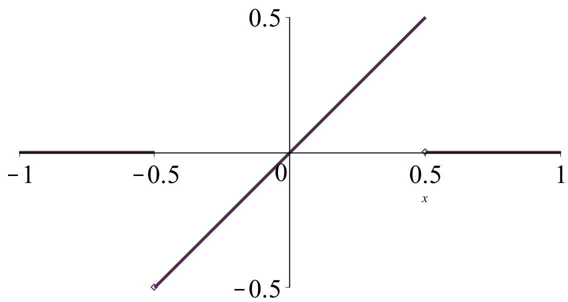



$f_o$

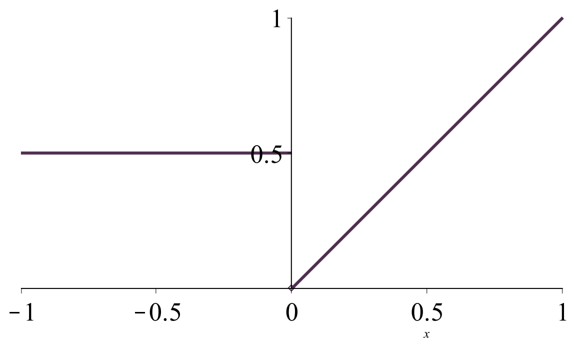



**b.** Find the cosine series expansion for $f$ (additional room to work is provided on the next page).

**7.** Consider the 2-periodic functions $f$ and $g$ shown below.




$y = f(x)$




$y = g(x)$

**a.** Find the Fourier series of $f$.

**b.** Describe the geometric operations that will transform the graph of $f$ into that of $g$. Use this to express $g(x)$ explicitly in terms of $f$.

**c.** By using part **b** (or directly), find the Fourier series of $g$. You may or may not find it useful to know that

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B),$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B).$$