Math 3357 Spring 2014

Partial Differential Equations

First Midterm Exam

Thursday, February 13

Your name (please print):

Instructions: This is a closed book, closed notes exam. You may use a calculator, **but the use of other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: ________________________________

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Points	14	12	14	14	8	10	14	14
Score								

Total:____________________

1. Solve the initial value problem

$$
\begin{aligned}
2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} &= x, \\
u(x, 0) &= x + 2.
\end{aligned}
$$

2. Solve the initial value problem

$$\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = u,$$
$$u(0,y) = \frac{1}{y^2+1}.$$

3. Consider the PDE

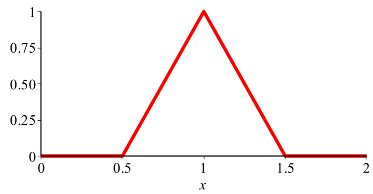
$$u_{xx} + 3u_{xy} + 2u_{yy} = 0. \tag{1}$$

a. If F and G are twice differentiable functions, show that $u(x,y) = F(2x-y) + G(x-y)$ is a solution to (1).

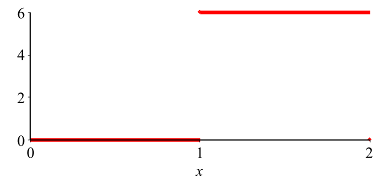
b. Use part **a** to find the solution to (1) that satisfies the initial conditions

$$u(x,0) = \frac{x}{x^2+1} \text{ and } u_y(x,0) = 0 \text{ for all } x.$$

4. Consider an ideal string with $L = 2$ and $c = 3$. Let f and g denote the functions whose graphs are shown below.



f

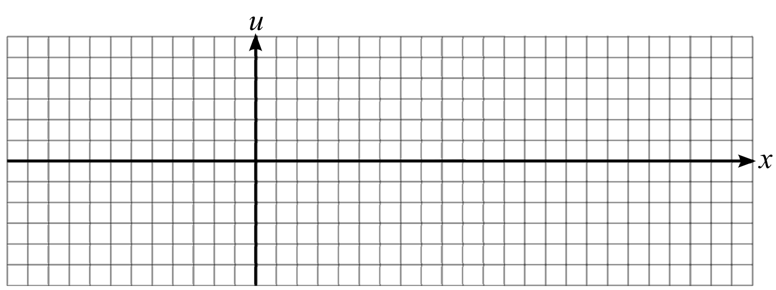


g

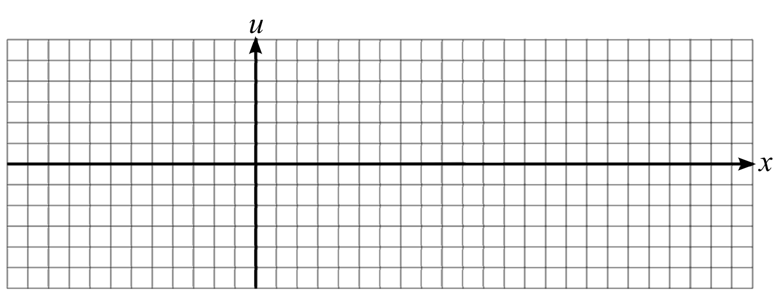
a. Once it begins moving, how long will it take for the string to return to its initial shape?

b. Carefully sketch the graphs of the 4-periodic odd extensions f^* and g^*, as well as an antiderivative G of g^*. Draw at least 2 periods of each.

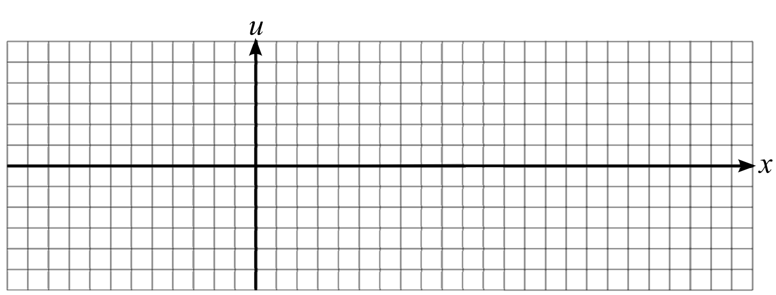
f^*



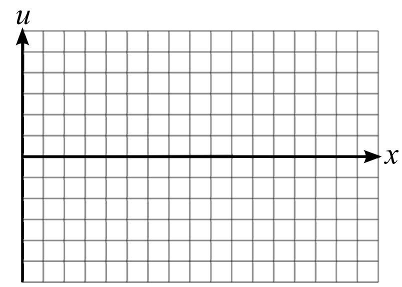
g^*



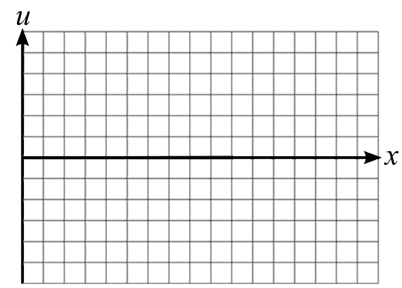
G



c. Suppose the string is stretched to have initial shape given by f and is released with zero initial velocity. Carefully sketch the shape of the string at $t = 1/6$.



d. Suppose the string is left in its rest position and is imparted with the initial velocity given by g. Carefully sketch the shape of the string at $t = 1/2$.



5. Determine the order of each of the following PDEs and state whether or not they are linear. If an equation is linear, state whether or not it is homogeneous.

a. $u_{xx} + (x^2 + y)u_{yy} = 0$

b. $u_x u_y + u_z = -u_{xyz}$

c. $(u_{xx} + u_x)_{tt} + e^{x+y}u_y = 0$

d. $\dfrac{\partial^2 u}{\partial x_1 \partial x_2} + \dfrac{\partial^2 u}{\partial x_2 \partial x_3} + \dfrac{\partial^2 u}{\partial x_1 \partial x_3} = u + x_1 x_2 x_3$

6. Show that the functions

$$\sin(x),\ \sin(3x),\ \sin(5x),\ \sin(7x), \ldots$$

form an orthogonal set on the interval $0 \leq x \leq \pi/2$. [*Remark:* You might find it useful to know that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$.]

7. Complete the following statement of the Fourier convergence theorem.

Theorem. *Suppose that f is a $2p$-periodic* ______________________________ *function. The Fourier series of f is given by*

$$a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\boxed{}\right) + b_n \sin\left(\boxed{}\right)\right)$$

where

$$a_0 = \boxed{}$$

and

$$a_n = \boxed{} \qquad b_n = \boxed{}$$

for $n \geq 1$. The Fourier series converges to

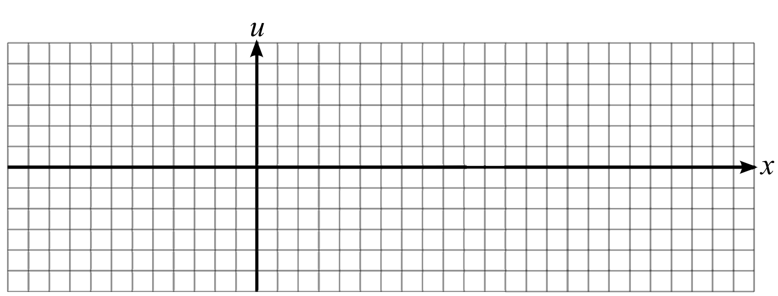
$$\boxed{}$$

for all x.

8. Consider the 2-periodic function satisfying

$$f(x) = \begin{cases} 1 & \text{if } -1 \leq x < 0, \\ x & \text{if } 0 \leq x < 1. \end{cases}$$

a. Carefully sketch the graph of the Fourier series of f (for at least two periods).



b. Find the Fourier series of f.