Math 3357 Spring 2015

Partial Differential Equations

Third Midterm Exam

Tuesday, April 27

Your name (please print):

Instructions: This is a closed book, closed notes exam. You may use a calculator, **but the use of other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: ______________________________

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**
Points	30	15	20	20	15
Score					

Total:________________________

1. Consider the second order linear ODE

$$(4 - x^2)y'' + 2y = 0. \tag{1}$$

a. Show that $x = 0$ is an ordinary point of (1).

b. Find the recurrence relation satisfied by the coefficients of the power series solutions to (1) that are centered at $x = 0$.

c. Find the first six terms (i.e. up to x^5) in each of two linearly independent power series solutions to (1) that are centered at $x = 0$.

d. Give a lower bound on the radius of convergence of every power series solution to (1) that is centered at $x = 0$.

2. Consider the second order linear ODE

$$x^2y'' - x(x+3)y' + (x+3)y = 0. \tag{2}$$

a. Show that $x = 0$ is a regular singular point of (2).

b. Determine the two possible values of r for which (2) may have a solution of the form $y = x^r \sum_{n=0}^{\infty} a_n x^n$. For which of these values is such a solution *guaranteed* to exist?

c. Find a lower bound for the radius of convergence of the analytic factor in the solution(s) in part **b.**

3. Consider the second order linear ODE boundary value problem

$$\begin{aligned} &xy'' + (1-x)y' + \left(\lambda - \frac{1}{x}\right)y = 0, \quad 0 < x < 1, \\ &y, y' \text{ bounded as } x \to 0^+, \quad y(1) = 0. \end{aligned} \tag{3}$$

a. Put (3) into Sturm-Liouville form. Identify the functions $p(x)$, $q(x)$ and $r(x)$. [*Suggestion:* First multiply through by a factor of the form e^{ax}.]

b. Give *two* reasons why this problem is *not* regular.

c. Compute $\langle x, e^x \rangle$, where $\langle \cdot, \cdot \rangle$ is the inner product associated with (3).

d. Show that eigenfunctions of (3) with distinct eigenvalues must be orthogonal (relative to the inner product in **b.**).

4. Use the method of Fourier transforms to solve

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0, \quad -\infty < x < \infty, \quad t > 0,$$
$$u(x,0) = f(x), \quad u_t(x,0) = 0.$$

5. Determine integers U, V and W so that

$$\int_0^{\alpha_{2n}} x^2 J_3(x)\,dx = U\alpha_{2n} J_1(\alpha_{2n}) + V J_0(\alpha_{2n}) + W.$$

[*Suggestion:* Use the identity $\int x^{-p+1} J_p(x)\,dx = -x^{-p+1} J_{p-1}(x) + C$.]