



MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 1.1
DUE JANUARY 23

Exercise 1. Use induction to prove that for any $n \in \mathbb{N}$,

$$\sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}.$$

Exercise 2. Given $n \in \mathbb{N}$ and $m \in \mathbb{Z}$, let $R(m)$ denote the remainder when m is divided by n . Prove that for all $a, b, c \in \mathbb{Z}$,

$$R(a + R(b + c)) = R(R(a + b) + c) \quad \text{and} \quad R(aR(bc)) = R(R(ab)c)$$