



**Exercise 1.** Let

$$G = \left\{ \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \pm \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \pm \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \right\}.$$

- a. Show that  $G$  is closed under matrix multiplication.
- b. Prove that  $G$  is a group under matrix multiplication. Is it abelian?

**Exercise 2.** Let  $G$  be a group and  $a, b, c \in G$ . Prove the following *cancellation laws*.

- a. If  $ab = ac$ , then  $b = c$  (*left cancellation*).
- b. If  $ab = cb$ , then  $a = c$  (*right cancellation*).

**Exercise 3.** Let  $G$  be a group and  $e_0 \in G$ . Given  $x, y \in G$ , define a new binary operation  $*$  on  $G$  by

$$x * y = xe_0^{-1}y.$$

Prove that  $G$  is a group under  $*$ .