MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 12.3
DUE MAY 1

Exercise 1. Let G be a group acting on a set S. Prove that for any $s \in S$,

$$\mathrm{Stab}(s) = \{x \in G \,|\, xs = s\}$$

is a subgroup of G.

Exercise 2. Let G be a group acting on a set S. Prove that if $s, t \in S$ and $t = xs$, then

$$\mathrm{Stab}(t) = x\,\mathrm{Stab}(s)x^{-1}.$$

Exercise 3. Recall that an action of a group G on a set S gives rise to a homomorphism $\pi : G \to \mathrm{Perm}(S)$, where $\pi_x(s) = xs$ for all $x \in G$ and $s \in S$. Prove that

$$\ker \pi = \bigcap_{s \in S} \mathrm{Stab}(s).$$