MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 2.2
DUE JANUARY 30

Exercise 1. Let G be a group and suppose that $x \in G$ has finite order n. Prove that

$$\{m \in \mathbb{Z} \,|\, x^m = e\} = n\mathbb{Z},$$

where $n\mathbb{Z} = \{nk \,|\, k \in \mathbb{Z}\}$.

Exercise 2. Let G be a finite group. Prove that if G has even order, then G contains an element with order 2.

Exercise 3. Let G be a group and $a, b \in G$. Denote the order of a by $|a|$. Prove the following assertions.

a. $|a| = |a^{-1}|$

b. $|ab| = |ba|$

c. $|a| = |bab^{-1}|$