Modern Algebra
Spring 2019

Assignment 4.1
Due February 13

Exercise 1. Prove that $Z(\mathrm{GL}_2(\mathbb{R})) = \{aI \mid a \in \mathbb{R}^\times\}$. [*Suggestion:* It is clear that every matrix of the form aI belongs to the center. To prove the opposite inclusion, commute an element of the center with the matrices $\left(\begin{smallmatrix} & 1 \\ 1 & \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1 & 1 \\ & 1 \end{smallmatrix}\right)$.]

Exercise 2. A group G is called *finitely generated* if there exist $x_1, x_2, \ldots, x_n \in G$ so that $G = \langle x_1, x_2, \ldots, x_n \rangle$.

a. Prove that $\mathbb{Z}^n = \underbrace{\mathbb{Z} \oplus \mathbb{Z} \oplus \cdots \oplus \mathbb{Z}}_{n \text{ times}}$ can be generated by n elements, and no fewer (this requires a little bit of linear algebra).

b. Prove that $\mathbb{Q}$ is *not* finitely generated. [*Suggestion:* Argue by contradiction.]