Exercise 1. Given $a, b \in \mathbb{Z}$, prove that

$$\langle a, b \rangle = \{ra + sb \mid r, s \in \mathbb{Z}\}.$$

This fact was tacitly assumed in our proof of Bézout's Lemma.

Exercise 2. Let $a, b \in \mathbb{Z}$. Use the classification of the subgroups of $\mathbb{Z}$ to prove that

$$a\mathbb{Z} \cap b\mathbb{Z} = \mathrm{lcm}(a, b)\mathbb{Z}.$$

Exercise 3. Use the fact that 1 has order 30 to compute the order of every element in $\mathbb{Z}_{30}$.