MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 5.1
DUE FEBRUARY 20

Exercise 1. Let G be the (additive) group of polynomials with real coefficients. For each $f \in G$, let $\int f$ denote the antiderivative of f whose graph passes through the point $(0,0)$. Show that the mapping $f \mapsto \int f$ from G to G is a homomorphism. What is its kernel? Can $(0,0)$ be replaced by any other point in $\mathbb{R}^2$?

Exercise 2. Define $f : D_4 \to \{\pm 1\}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a rotation,} \\ -1 & \text{otherwise.} \end{cases}$$

Prove that f is a homomorphism. What is its kernel?

Exercise 3. Let A be an (additive) abelian group and $B, C < A$. Define $f : B \times C \to A$ by $f(b,c) = b + c$.

a. Prove that f is a homomorphism.

b. Use part **a** to prove that $B + C = \{b + c \,|\, b \in B, c \in C\}$ is a subgroup of A.

c. Compute $\ker f$.