Exercise 1. Show that every coset of $\mathbb{R}^+$ in $\mathbb{C}^\times$ contains a unique element of absolute value equal to 1. Conversely, show that every coset of $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$ in $\mathbb{C}^\times$ contains a unique positive real number. In both cases, describe the cosets geometrically.

Exercise 2. Let G be a group and $S \subset G$. Given $x, y \in G$, define $x \sim y$ by the condition $xy^{-1} \in S$. Prove that if $\sim$ is an equivalence relation on G, then $S < G$.

Exercise 3. Prove that $\mathbb{Q}$ has an uncountable number of cosets in $\mathbb{R}$. [*Suggestion:* Argue by contradiction.]