MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 7.1
DUE MARCH 6

Exercise 1. Let G be a group and $H < G$. Prove that if $[G : H] = 2$, then $H \lhd G$.

Exercise 2. Let G, H be groups, $f : G \to H$ a homomorphism.

a. If $K \lhd H$, prove that $f^{-1}(K) \lhd G$.

b. If $K \lhd G$ and f is surjective, prove that $f(K) \lhd H$.

Exercise 3. Let G be a group, $H < G$ and $N \lhd G$.

a. Prove that $HN < G$.

b. Prove that $H \cap N \lhd H$.

c. Prove that if $H \lhd G$ and $H \cap N$ is trivial, then $hn = nh$ for all $h \in H$ and $n \in N$.