Modern Algebra
Spring 2019

Assignment 7.3
Due March 6

**Exercise 1.** Let $G$ be a group and $H < G$. The *normalizer of* $H$ *in* $G$ is

$$N_G(H) = \{x \in G \mid xHx^{-1} = H\}.$$

**a.** Prove that $N_G(H)$ is a subgroup of $G$ containing $H$, and that $H$ is normal in $N_G(H)$.

**b.** Prove that the set $\{xHx^{-1} \mid x \in G\}$ of conjugates of $H$ is in one to one correspondence with the left cosets of $N_G(H)$ in $G$.

**Exercise 2.** Let $G$ be a group and $H < G$. The *centralizer of* $H$ *in* $G$ is

$$Z_G(H) = \{x \in G \mid xy = yx \text{ for all } y \in H\}.$$

Prove that $Z_G(H)$ is a normal subgroup of $N_G(H)$.

**Exercise 3.** Let $G$ be a group, $H < G$. Recall the homomorphism $T : G \to \mathrm{Perm}(G/H)$ of Lang, II.4.28, which is defined by $x \mapsto T_x$, where $T_x(yH) = xyH$. Prove that

$$\ker T = \bigcap_{x \in G} xHx^{-1},$$

the so-called *normal core* of $H$.