Modern Algebra
Spring 2019

Assignment 8.2
Due March 20

Exercise 1. Let $f : X \to Y$ be a function.

a. Let $A \subset X$. Prove that $A \subset f^{-1}(f(A))$, with equality if f is injective.

b. Let $B \subset Y$. Prove that $f(f^{-1}(B)) \subset B$, with equality if f is surjective.

Exercise 2. Let a and b be elements of a group G. If a has odd order and $aba^{-1} = b^{-1}$, prove that $|b| = 2$. [*Suggestion:* Notice that the hypotheses imply that conjugation by a is an automorphism of $\langle b \rangle$ of order 2.]

Exercise 3. Let G be a finite abelian group, $n \in \mathbb{N}$, and suppose that $\gcd(|G|, n) = 1$. Prove that every element of G has a unique nth root. [*Suggestion:* Reformulate the conclusion in terms of the map $x \mapsto x^n$.]