MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 8.3
DUE MARCH 20

Exercise 1. Recall

$$\mathbb{Z}_{(p)} = \left\{ \frac{r}{s} \,\middle|\, r \in \mathbb{Z}, s \in \mathbb{N}, p \nmid s \right\},$$

which is a subgroup of $\mathbb{Q}$.

a. Show that for any $k \in \mathbb{N}$,

$$p^k \mathbb{Z}_{(p)} = \{p^k a \mid a \in \mathbb{Z}_{(p)}\}$$

is a subgroup of $\mathbb{Z}_{(p)}$.

b. Inclusion composed with the canonical epimorphism yields a homomorphism

$$f : \mathbb{Z} \to \mathbb{Z}_{(p)}/p^k \mathbb{Z}_{(p)}.$$

Prove that f is surjective and use the First Isomorphism Theorem to prove that

$$\mathbb{Z}/p^k\mathbb{Z} \cong \mathbb{Z}_{(p)}/p^k \mathbb{Z}_{(p)}.$$