Exercise 1. Express each of the following permutations as products of disjoint cycles (you may feel free to omit 1-cycles) and as products of transpositions.

a. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 4 & 2 & 8 & 7 & 6 & 3 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 9 & 1 & 5 & 2 & 7 & 8 & 6 & 4 & 3 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 8 & 1 & 9 & 5 & 4 & 2 & 3 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 11 & 9 & 10 & 8 & 6 & 7 & 4 & 5 & 3 \end{pmatrix}$

Exercise 2. Let $\sigma \in S_n$ and suppose $\sigma = c_1 c_2 \cdots c_k$, where $c_1, c_2, \ldots, c_k$ are disjoint cycles.

a. Explain why $\sigma^m = \mathrm{Id}$ if and only if $c_i^m = \mathrm{Id}$ for all i.

b. If $\ell(c)$ denotes the length of a cycle c, prove that

$$|\sigma| = \mathrm{lcm}(\ell(c_1), \ell(c_2), \ldots, \ell(c_k)).$$

Exercise 3. Determine the order of each permutation in Exercise 1.