INTRO TO ABSTRACT MATHEMATICS — ASSIGNMENT 3.2
SPRING 2020 — DUE FEBRUARY 12

Exercise 1. Let $a, b \in \mathbb{N}$. The *greatest common divisor* of a and b, denoted $\gcd(a, b)$, is defined to be the greatest element in the set $\{d \in \mathbb{N} \mid d \text{ divides both } a \text{ and } b\}$. Prove that $\gcd(a, b) = \gcd(a+b, a)$. [*Suggestion:* Show that the sets defining the two gcds are the same.]

Exercise 2. The *symmetric difference* of two sets A and B is defined to be

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

Verify the following properties of the symmetric difference.

a. $A \Delta B = B \Delta A$.

b. $A \Delta B = (A \cup B) \setminus (A \cap B)$.

c. $A \Delta B = A \cup B$ if and only if A and B are disjoint.

Exercise 3. Let A, B, C be sets.

a. Show that

$$(A \Delta B) \Delta C = [(A \cup B \cup C) \setminus ((A \cap B) \cup (A \cap C) \cup (B \cap C))] \cup (A \cap B \cap C).$$

b. Conclude that $(A \Delta B) \Delta C = (B \Delta C) \Delta A$.

c. Use **2a** and part **b** to show that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$.