Intro to Abstract Mathematics
Spring 2020

Assignment 5.2
Due February 28

Exercise 1. Let $a, b \in \mathbb{R}$. Prove that if $a < b$, then $a < \frac{a+b}{2} < b$.

Exercise 2. Let $x, y, z \in \mathbb{Z}^+$. Show that if $xy = z$, then $x^2 \leq z$ or $y^2 \leq z$.

Exercise 3.

a. Let P, Q, R be statements. Prove that $P \to (Q \vee R) \cong (P \wedge \neg R) \to Q$.

b. Prove that if $n \in \mathbb{Z}$, then n is even or $n + 1$ is even.

Exercise 4. Let $f(n) = n^2 + n + 41$. Prove or disprove the following statement:

If $n \in \mathbb{N}$, then $f(n)$ is prime.