Intro to Abstract Mathematics
Spring 2020

Assignment 7.2
Due March 18

Exercise 1. Prove that for all $a, b, c \in \mathbb{Z}$, if $c|a$ and $c|b$, then $c|(xa + yb)$ for all $x, y \in \mathbb{Z}$.

Exercise 2. Let $a, b \in \mathbb{Z}$ and suppose that there exist $x, y \in \mathbb{Z}$ so that $xa + yb = 1$. Prove that 1 is the only common divisor of a and b.

Exercise 3. Let $a, b, c \in \mathbb{Z}$. Prove that if a divides either b or c, then a divides bc. Is the converse true? Justify your answer.

Exercise 4. Let $A = \{a \in \mathbb{N} \,|\, a \text{ is odd}\}$, $B = \{x^2 + y^2 \,|\, x, y \in \mathbb{N}\}$ and $C = \{4k + 1 \,|\, k \in \mathbb{N}\}$. Prove that $A \cap B \subset C$. Are the sets equal? Justify your answer. [*Suggestion:* Notice that if a is odd and $a = x^2 + y^2$, then one of x^2 and y^2 must be even, and the other must be odd.]