



INTRO TO ABSTRACT MATHEMATICS  
SPRING 2020

ASSIGNMENT 8.1  
DUE APRIL 1

**Exercise 1.** Let  $m \in \mathbb{N}^+$ . Suppose  $n \in \mathbb{Z}$  and  $n < 0$ . Show that there exist  $q, r \in \mathbb{Z}$  so that

$$n = qm + r \quad \text{and} \quad 0 \leq r < m.$$

[*Suggestion:* Apply the division algorithm to  $-n$  or  $-n - 1$ .]

**Exercise 2.** Let  $m, n \in \mathbb{N}^+$ . Use the division algorithm to write  $n = qm + r$  with  $q, r \in \mathbb{N}$  and  $0 \leq r < m$ . Prove that  $\gcd(m, n) = \gcd(r, m)$ .<sup>1</sup> [*Suggestion:* Proceed as in Exercise 3.2.1.]

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<sup>1</sup>This is the basis of the *Euclidean algorithm*, the most efficient way known to compute greatest common divisors.