INTRO TO ABSTRACT MATHEMATICS | ASSIGNMENT 9.1
SPRING 2020 | DUE APRIL 8

Exercise 1. Consider the sequence $a_0, a_1, a_2, \ldots$ of real numbers defined by

$$a_0 = 0,$$
$$a_{n+1} = a_n^2 + \frac{1}{4} \text{ for all } n \in \mathbb{N}.$$

a. Compute a_1, a_2 and a_3.

b. Use induction to prove that $0 < a_n < 1/2$ for all $n \geq 1$.

c. Prove that $a_{n+1} > a_n$ for all $n \in \mathbb{N}$. [*Suggestion:* First prove that $x^2 + 1/4 > x$ for all real $x \neq 1/2$.]

Exercise 2. Prove that for all $n \geq 1$, $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}$.