Complex Variables
Spring 2020

Assignment 1.1
Due January 29

Exercise 1. If $z \in \mathbb{C}^\times$, prove that the equation $w^2 = z$ has exactly two solutions in $\mathbb{C}$.[1] Express $\operatorname{Re} w$ and $\operatorname{Im} w$ in terms of $\operatorname{Re} z$ and $\operatorname{Im} z$.

Exercise 2. For $x, y \in \mathbb{R}$, the *complex conjugate* of $z = x+iy \in \mathbb{C}$ is defined to be $\overline{z} = x-iy$. Prove that the map $z \mapsto \overline{z}$ is a field automorphism of $\mathbb{C}$ with fixed field $\mathbb{R}$.[2] What is its inverse?

Exercise 3. If $z \in \mathbb{C}^\times$, invert the matrix form of z to find expressions for $\operatorname{Re}(1/z)$ and $\operatorname{Im}(1/z)$ in terms of $\operatorname{Re} z$ and $\operatorname{Im} z$.

Exercise 4. Show that the eigenvalues of the matrix form of $z \in \mathbb{C}$ are precisely z and $\overline{z}$.

[1] That is, every nonzero complex number has exactly two square roots.

[2] If $\sigma : F \to F$ is a field automorphism, the elements $a \in F$ satisfying $\sigma(a) = a$ form a subfield of F called the *fixed field* of σ.