



COMPLEX VARIABLES  
SPRING 2020

ASSIGNMENT 10  
DUE APRIL 15

**Exercise 1.** Textbook exercise IV.4.1.

**Exercise 2.** Prove that  $\int_0^\pi e^{\cos \theta} \cos(\sin \theta) d\theta = \pi$  by considering  $\int_\gamma (e^z/z) dz$ , where  $\gamma$  is the unit circle.

**Exercise 3.** Textbook exercise IV.4.2.

**Exercise 4.** Let  $f$  be analytic on a disk  $D$  and let  $\gamma$  be any simple loop in  $D$ . Prove that

$$\int_\gamma \frac{f(z)}{(z - z_0)^2} dz = \int_\gamma \frac{f'(z)}{(z - z_0)} dz$$

for any  $z_0 \in D \setminus \gamma$ . Generalize.

**Exercise 5.** Suppose that  $f$  is analytic on  $|z| < 1$  and that it satisfies the inequality  $|f(z)| \leq 1$ . Prove that  $|f'(0)| \leq 1$ .