Complex Variables
Spring 2020

Assignment 2.1
Due February 5

Exercise 1. Use de Moivre's formula and the binomial theorem to prove that

$$\cos(n\theta) = \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} (-1)^k \cos^{n-2k}\theta \sin^{2k}\theta$$

for all $n \in \mathbb{N}$.

Exercise 2. In this exercise we will give an alternate proof of the triangle inequality.

a. Prove that the triangle inequality is equivalent to the statement that $|1 + re^{i\theta}| \leq 1 + r$ for all $\theta \in \mathbb{R}$ and $r \geq 0$. [*Suggestion:* Use polar coordinates.]

b. Without using the triangle inequality, prove that $|1 + re^{i\theta}| \leq 1 + r$ for all $\theta \in \mathbb{R}$ and $r \geq 0$. [*Suggestion:* Consider $|1 + re^{i\theta}|^2$.]

Exercise 3. Textbook exercise I.2.7.