Complex Variables
Spring 2020

Assignment 3.1
Due February 12

Exercise 1. Show that for $z \in \mathbb{C}$ the principal value of $\sqrt{z}$ is given by

$$\sqrt{\frac{|z| + \operatorname{Re} z}{2}} + i \operatorname{sgn}(\operatorname{Im} z)\sqrt{\frac{|z| - \operatorname{Re} z}{2}},$$

where $\operatorname{sgn}(\cdot)$ is the sign function.

Exercise 2. Give necessary and sufficient conditions on $z, z_1, z_2 \in \mathbb{C}$ so that the following equations hold.

a. $\sqrt{z_1 z_2} = \sqrt{z_1}\sqrt{z_2}$.

b. $\sqrt{z^2} = z$.

Exercise 3. Let $a \in \mathbb{R}^+$. For $z \in \mathbb{C} \setminus \{\pm a\}$, let

$$f(z) = \sqrt{z+a}\sqrt{z-a}.$$

Show that $f(z)$ defines a branch of $\sqrt{z^2 - a^2}$ that is continuous on $\Omega = \mathbb{C}\setminus[-a, a]$. Determine $f(x + i0)$ and $f(x - i0)$ for $x \in (-a, a)$. [*Suggestion:* In light of what we know about the function $\sqrt{z}$, it suffices to show that $f(x + i0) = f(x - i0)$ for $x \in (-\infty, -a)$.]