COMPLEX VARIABLES
SPRING 2020

ASSIGNMENT 5.2
DUE FEBRUARY 26

Exercise 1. Let $a \in \mathbb{C}^\times$. Prove that the principal branch of $f(z) = z^a$ is analytic on $\mathbb{C} \setminus (-\infty, 0]$ with $f'(z) = az^{a-1}$.

Exercise 2. Prove that, when extended to complex-valued functions, the differential operators $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ are $\mathbb{C}$-linear and obey the product rule.

Exercise 3. Use the preceding exercise to show that the operators $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial \bar{z}}$ are $\mathbb{C}$-linear and obey the product rule.

Exercise 4. Let $\Omega \subset \mathbb{C}$ be a domain and let $D : C^1(\Omega) \to C^0(\Omega)$ be a $\mathbb{C}$-linear operator that obeys the product rule. Prove that for any $f \in C^1(\Omega)$ and any $m \in \mathbb{N}$ one has $D(f^m) = mf^{m-1}D(f)$. [*Suggestion:* Induct on m.]