Exercise 1. Let $U, V \subset \mathbb{C}$ be domains, let $f : U \to \mathbb{C}$ be analytic with $f(U) \subset V$, and let $u : V \to \mathbb{R}$ be harmonic. Prove that $u \circ f : U \to \mathbb{R}$ is harmonic.

Exercise 2. Let $z, w \in \mathbb{C}$. Show that, when viewed as vectors in $\mathbb{R}^2$, one has

$$z \cdot w = \frac{\overline{z}w + z\overline{w}}{2} \quad \text{and} \quad z \times w = \frac{\overline{z}w - z\overline{w}}{2i}.$$

Conclude that the complex product can be expressed in terms of the dot and cross products:

$$zw = \overline{z} \cdot w + i(\overline{z} \times w)$$

Exercise 3. Let $f = u + iv$ be an analytic function. Let $k = u(z_0)$ and $\ell = v(z_0)$. Prove that if $f'(z_0) \neq 0$, then the contours $u(x, y) = k$ and $v(x, y) = \ell$ are orthogonal at z_0.

Exercise 4. Find an expression for the cross-ratio $[z_0, z_1, z_2, z_3]$ when $z_j = \infty$, for $j = 0, 1, 2, 3$.

Exercise 5. Textbook exercise II.7.1.

Exercise 6. Textbook exercise II.7.9.