



COMPLEX VARIABLES
SPRING 2020

ASSIGNMENT 7.1
DUE MARCH 18

Exercise 1. Show that the FLT $f(z) = \frac{1+z}{1-z}$ maps the upper half of the unit disk $\{z \in \mathbb{C} \mid |z| < 1, \operatorname{Im} z > 0\}$ conformally onto the first quadrant.

Exercise 2. Let $f(z) = \frac{z-i}{z+i}$. What is the image under f of

- a. the real line?
- b. the circle with center 0 and radius 2?
- c. the unit circle?
- d. the imaginary axis?

Exercise 3. Find a conformal map that maps the quarter disk $\{z \in \mathbb{C} \mid 0 < |z| < 1, 0 < \arg z < \pi/2\}$ onto the open unit disk. [*Suggestion:* Compose several simpler maps. Exercises 1 and 2 may be helpful.]

Exercise 4. Suppose $a, b, c, d \in \mathbb{R}$ and $ad - bc > 0$. Prove that $f(z) = \frac{az+b}{cz+d}$ maps the upper half-plane $H = \{z \in \mathbb{C} \mid \operatorname{Im} z > 0\}$ onto itself. [*Suggestion:* Compute $\operatorname{Im} f(x + iy)$.]