COMPLEX VARIABLES
SPRING 2020

ASSIGNMENT 8.2
DUE APRIL 1

Exercise 1. Textbook exercise III.5.2.

Exercise 2. Let $f(z)$ be analytic and bounded on the right half-plane $\operatorname{Re} z > 0$. Suppose $f(z)$ extends continuously to the imaginary axis and satisfies $|f(iy)| \le M$ for all $y \in \mathbb{R}$. We will show that $|f(z)| \le M$ throughout $\operatorname{Re} z > 0$.

Fix $\epsilon > 0$ and let $g_\epsilon(z) = \dfrac{f(z)}{(1+z)^\epsilon}$, where we are using the principal branch of w^ϵ. For $R > 1$ let Ω_R denote $\{\operatorname{Re} z > 0\} \cap \{|z+1| < R\}$ and let $C_R = \partial\Omega_R$.

- **a.** Show that if R is sufficiently large, then $|g_\epsilon(z)| \le M$ on C_R. Conclude that $|g_\epsilon(z)| \le M$ on Ω_R, when R is large enough.
- **b.** Use part **a** to prove that $|g_\epsilon(z)| \le M$ everywhere on the right half-plane $\operatorname{Re} z > 0$.
- **c.** Use part **b** to show that $|f(z)| \le M$ throughout the half-plane, by letting $\epsilon \to 0^+$ pointwise.