Complex Variables
Spring 2020

Assignment 8.3
Due April 1

Exercise 1. Recall that for $h : [a, b] \to \mathbb{C}$ we defined

$$\int_a^b h(t)\, dt = \int_a^b \operatorname{Re} h(t)\, dt + i \int_a^b \operatorname{Im} h(t)\, dt.$$

If $\alpha \in \mathbb{C}$, use this definition to prove that

$$\int_a^b \alpha h(t)\, dt = \alpha \int_a^b h(t)\, dt.$$

Exercise 2. Let $\Omega \subset \mathbb{C}$ be a domain, $f : \Omega \to \mathbb{C}$ be continuous, and set $u = \operatorname{Re} f$, $v = \operatorname{Im} f$. If $\gamma(t)$, $t \in [a, b]$, is a C^1 path in Ω, prove that

$$\int_\gamma u\, dx - v\, dy + i \int_\gamma v\, dx + u\, dy = \int_a^b f(\gamma(t))\, \gamma'(t)\, dt.$$

Exercise 3. Textbook exercise IV.1.1.

Exercise 4. Textbook exercise IV.1.4. [*Suggestion:* Use Green's theorem]

Exercise 5. Textbook exercise IV.1.5.