Exercise 1. Let $\alpha = a + bi \in \mathbb{C}$. Define $T_\alpha : \mathbb{C} \to \mathbb{C}$ by $T_\alpha(z) = \alpha z$.

- **a.** Prove that T_α is an $\mathbb{R}$-linear transformation.
- **b.** Find the matrix for T_α relative to the basis $\mathcal{B} = \{1, i\}$.
- **c.** Suppose $\alpha \in \mathbb{R}$. Show that $[T_\alpha]$ is a scalar matrix.
- **d.** Suppose $|\alpha| = \sqrt{a^2 + b^2} = 1$. Show that $[T_\alpha]$ is a rotation matrix.
- **e.** Suppose $\alpha \neq 0$. Since $\alpha = |\alpha| \cdot \frac{\alpha}{|\alpha|}$, it follows that $T_\alpha = T_{|\alpha|} \circ T_{\alpha/|\alpha|}$. Use this fact and parts **c** and **d** to give a geometric description of multiplication by α.

Exercise 2. Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{M}_2(\mathbb{R})$$

be a fixed 2×2 matrix and define $T_A : \mathrm{M}_2(\mathbb{R}) \to \mathrm{M}_2(\mathbb{R})$ by $T_A(B) = AB$.

- **a.** Show that T_A is a linear transformation.
- **b.** Let $\mathcal{B} = \{E_{11}, E_{12}, E_{21}, E_{22}\}$, where E_{ij} has a 1 in the ij-entry, and zeros elsewhere. Find the matrix for T_A relative to $\mathcal{B}$.
- **c.** Generalize part **b** to square matrices of arbitrary size, i.e. take $A \in \mathrm{M}_n(\mathbb{R})$ instead.