Linear Algebra
Assignment 7.1
Spring 2021
2.2. \# 20, 31, 33-35

Exercise 1. Let $A, B$ be a $n \times n$ matrices, and suppose that $A$ is invertible. Prove that $B$ is invertible if and only if $A B A^{-1}$ is invertible, and that in this case we have $\left(A B A^{-1}\right)^{-1}=$ $A B^{-1} A^{-1}$.

Exercise 2. Let $A, B$ be $n \times n$ matrices.
a. Suppose that $A B=I$. Use the Invertible Matrix Theorem to show that $A$ is invertible, and that $B=A^{-1}$.
b. Suppose that $B A=I$. Use the Invertible Matrix Theorem to show that $A$ is invertible, and that $B=A^{-1}$.

This shows that for square matrices, the existence of a one-sided inverse (on either side) is equivalent to the existence of a two-sided inverse. Practically speaking, this means that in order to check that $B=A^{-1}$, we need only show that $A B=I$ or $B A=I$.

