Linear Algebra
Spring 2021

Assignment 7.2
Due March 24

Exercise 1. For the following matrices A, B, compute AB using the right multiplication law.

a. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$

b. $A = \begin{pmatrix} -4 & 0 & 1 \\ 2 & -2 & 1 \\ 3 & 1 & 7 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 \\ 1 & 0 \\ 1 & -3 \end{pmatrix}$

c. $A = \begin{pmatrix} 1 & 0 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \\ 6 & -1 \end{pmatrix}$

Exercise 2.[1] Every plane P through the origin in $\mathbb{R}^3$ is given by an equation of the form

$$0 = ax + by + cz = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{n}^T \mathbf{x},$$

where

$$\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq 0.$$

Geometrically, $\mathbf{n}$ is perpendicular to P (a *normal vector* to P), and if we require $\mathbf{n}$ to be a unit vector, then the association $\mathbf{n} \mapsto P$ is a two-to-one correspondence between the points on the unit sphere S^2 and the planes through $\mathbf{0}$ in $\mathbb{R}^3$ (why is it not one-to-one?).

Here's the exercise. Let P be a plane through $\mathbf{0}$ in $\mathbb{R}^3$ given by $ax + by + cz = 1$ with unit normal $\mathbf{n} = \begin{pmatrix} a & b & c \end{pmatrix}^T$, and let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the reflection through P, which is a linear transformation (why?). Choose "convenient" vectors $\mathbf{v}_1, \mathbf{v}_2$ in P so that $\mathcal{B} = \{\mathbf{n}, \mathbf{v}_1, \mathbf{v}_2\}$ is a basis for $\mathbb{R}^3$, and compute $[T]$ from $\begin{pmatrix} T(\mathbf{n}) & T(\mathbf{v}_1) & T(\mathbf{v}_2) \end{pmatrix}$ and $\begin{pmatrix} \mathbf{n} & \mathbf{v}_1 & \mathbf{v}_2 \end{pmatrix}$, in terms of a, b, c alone.

[1] This exercise might be somewhat challenging. Once we define *orthogonal projection*, the matrix $[T]$ will be easy to compute. In the mean time, I'm fairly certain the computation of $[T]$ outlined in this exercise is tractable, but I haven't actually worked out the details.