Number Theory
Spring 2022

Assignment 2
Due February 16

Exercise 1. Let $a \in \mathbb{Z}$. Show that for any $n \in \mathbb{Z}$, $(a, a+n)$ divides n. Conclude that $(a, a+1) = 1$.

Exercise 2. If $a, b \in Z$ are not both zero, prove that $(2a - 3b, 4a - 5b)$ divides b. Conclude that $(2a + 3, 4a + 5) = 1$.

Exercise 3. Use the Euclidean Algorithm to find integers r and s satisfying the following:

a. $(24, 138) = 24r + 138s$.

b. $(119, 272) = 119r + 272s$.

c. $(1769, 2378) = 1769r + 2378s$.

Exercise 4. Let $a, b \in \mathbb{Z}$. Prove that $(a, b) = 1$ if and only if there exist $x, y \in \mathbb{Z}$ so that $ax + by = 1$.

Exercise 5. If $a, b, n \in \mathbb{N}$, prove that $(a, b) = 1$ if and only if $(a^n, b^n) = 1$. [*Suggestion:* Use the preceding exercise.]