Partial Differential Equations | Assignment 3.2
Spring 2023 | Due January 31

Exercise 1. Textbook exercise 2.1.15 [*Suggestion:* Consider $F(x+2\pi) - F(x)$.]

Exercise 2. Textbook exercises 2.2.1–2.2.4

Given (integrable) functions f and g on the interval $[a, b]$, recall that we defined their *inner product* to be

$$\langle f, g\rangle = \int_a^b f(x)g(x)\, dx.$$

We say f and g are *orthogonal* (on $[a, b]$) if $\langle f, g\rangle = 0$.

Exercise 3. The *Legendre polynomials* $P_n(x)$ are defined recursively by

$$P_0(x) = 1, \quad P_1(x) = x,$$
$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x) \quad \text{for } n \geq 1.$$

a. Compute $P_2(x)$, $P_3(x)$ and $P_4(x)$.

b. Show that P_0, P_1, P_2, P_3 and P_4 are pairwise orthogonal on the interval $[-1, 1]$.

c. Compute $\langle P_n, P_n\rangle$ for $n = 0, 1, 2, 3, 4$, the inner product being taken over $[-1, 1]$.

Exercise 4. Show that any (integrable) even function is orthogonal to any (integrable) odd function on $[-a, a]$.