Modern Algebra
Spring 2023

Assignment 1.1
Due January 18

Exercise 1. Let G be a group. Show that for all $a, b, c \in G$, $ac = bc$ implies $a = b$. This is known as the *right cancellation law*. Show that the analogous *left cancellation law* holds in G as well.

Exercise 2. If S is a set and $\circ$ denotes composition of functions, prove that $(\mathrm{Perm}(S), \circ)$ is a group. Show that $\mathrm{Perm}(S)$ is abelian if and only if $|S| \leq 2$.

Exercise 3. Let G be a group. Given $a \in G$, define $\lambda_a : G \to G$ by $\lambda_a(g) = ag$ for all $g \in G$. The function λ_a is sometimes called *(left) translation by a*.

a. Prove that for any $a \in G$ one has $\lambda_a \in \mathrm{Perm}(G)$.

b. Prove that for all $a, b \in G$, $\lambda_a = \lambda_b$ if and only if $a = b$. [*Suggestion:* Evaluate both functions at the identity.]