MODERN ALGEBRA
SPRING 2019

ASSIGNMENT 10.1
DUE MARCH 29

Exercise 1. Let $\gamma = (i_1\, i_2\, \cdots\, i_r) \in S_n$ be an r-cycle.

a. Prove that $\epsilon(\gamma) = (-1)^{r+1}$.

b. If $\sigma \in S_n$, show that

$$\sigma(i_1\, i_2\, \cdots\, i_r)\sigma^{-1} = (\sigma(i_1)\, \sigma(i_2)\, \cdots\, \sigma(i_r)).$$

Exercise 2. Determine the parity of each of the following permutations.

a. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 4 & 2 & 8 & 7 & 6 & 3 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 9 & 1 & 5 & 2 & 7 & 8 & 6 & 4 & 3 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 8 & 1 & 9 & 5 & 4 & 2 & 3 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 11 & 9 & 10 & 8 & 6 & 7 & 4 & 5 & 3 \end{pmatrix}$

Exercise 3. Let G be a finite group of order n. Given $a \in G$, let $\lambda_a \in \mathrm{Perm}(G) \cong S_n$ denote left translation by a. If a has order m, prove that $\epsilon(\lambda_a) = (-1)^{n(m+1)/m}$. [*Suggestion:* Show that the cycles of λ_a are the right cosets of $\langle a \rangle$ in G.]