Modern Algebra
Spring 2023

Assignment 10.2
Due March 29

Exercise 1. Let G be a group. Given $a, b \in G$ we say that a is *conjugate* to b provided there is an $x \in G$ so that $xax^{-1} = b$. Prove that conjugacy (being conjugate to) defines an equivalence relation on G. The equivalence classes under this relation are called the *conjugacy classes* of G.

Exercise 2. Prove that $H = \{\mathrm{id}, (12)(34), (13)(24), (14)(23)\}$ is a normal subgroup of S_4. Since $H \leq A_4$, this proves that $H \triangleleft A_4$, and hence that A_4 is *not* simple. Prove that $S_4/H \cong S_3$ and $A_4/H \cong \mathbb{Z}_3$.

Exercise 3. Carefully prove that any two r-cycles are conjugate in S_n (the proof I gave in class wasn't quite correct).