MODERN ALGEBRA
SPRING 2023

ASSIGNMENT 13.2
DUE APRIL 19

Exercise 1. Let A be an additive abelian group and for $k \in \mathbb{N}$ let C_k denote a cyclic subgroup of A with order k.

a. Let $m, n \in \mathbb{N}$ and suppose $\gcd(m, n) = 1$. Use Lagrange's Theorem to show that the sum $C_m + C_n$ is direct.

b. With m and n as above, write $C_m = \langle a \rangle$ and $C_n = \langle b \rangle$. Show that $a + b$ has order mn. Conclude that $C_m \oplus C_n = C_{mn}$.

c. Let $m_1, m_2, \ldots, m_k \in \mathbb{N}$ and suppose $\gcd(m_i, m_j) = 1$ for all $i \neq j$. Use induction to prove that

$$C_{m_1} \oplus \cdots \oplus C_{m_k} = C_{m_1 m_2 \cdots m_k}.$$

Exercise 2. Classify the finite abelian groups of order 4312 (there are 6, up to isomorphism).

Exercise 3. Lang, exercise II.7.2.