MODERN ALGEBRA
SPRING 2023

ASSIGNMENT 14.1
DUE APRIL 26

Exercise 1. Let A and B be abelian groups, let $i_A : A \to A \oplus B$ be defined by $i_A(a) = (a, 0)$, and let $i_B : B \to A \oplus B$ be given by $i_B(b) = (0, b)$. Let $\widehat{i_A} : \widehat{A \oplus B} \to \widehat{A}$ and $\widehat{i_B} : \widehat{A \oplus B} \to \widehat{B}$ be the dual maps. Show that the map

$$\widehat{i_A} \oplus \widehat{i_B} : \widehat{A \oplus B} \to \widehat{A} \oplus \widehat{B}$$

defined by $\chi \mapsto \left(\widehat{i_A}(\chi), \widehat{i_B}(\chi)\right)$ is a homomorphism.

Exercise 2. Let $n \in \mathbb{N}$. Define $f : \widehat{\mathbb{Z}/n\mathbb{Z}} \to \boldsymbol{\mu}_n$ by $f(\chi) = \chi(1)$, as in class. Prove that f is a homomorphism.