



MODERN ALGEBRA
SPRING 2023

ASSIGNMENT 2.2
DUE JANUARY 25

Exercise 1. Construct Cayley tables for the dihedral groups D_3 and D_4 .

Exercise 2. Given $n \geq 3$, let $P \subset \mathbb{R}^2$ be a regular n -gon. Give each vertex of P a unique label (in any way you like). Let $r : P \rightarrow P$ denote the smallest possible counterclockwise rotation of P about its center, and let $f : P \rightarrow P$ denote any symmetry of P that reverses the ordering of the labels of the vertices of P .

- Explain why $D_n = \{\text{id}, r, r^2, \dots, r^{n-1}, f, rf, r^2f, \dots, r^{n-1}f\}$.
- Show that $fr = r^{n-1}f$.
- Show that D_n is nonabelian by proving that $rf \neq fr$.

Exercise 3. Describe the group of symmetries of the doubly infinite string

$\dots \text{HHHH} \dots$

Is this group abelian?

Exercise 4. How large is the group of symmetries of a regular tetrahedron? Is it abelian?