MODERN ALGEBRA
SPRING 2023

ASSIGNMENT 4.2
DUE FEBRUARY 9

Exercise 1.

Let $a, b, c \in Z$ with $d = \gcd(a, b) \neq 0$, so that $\frac{a}{d}, \frac{b}{d} \in \mathbb{Z}$. Use Bézout's Lemma to prove the following.

a. $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. [*Suggestion.* Apply Bézout's Lemma to $\gcd(a, b)$, then divide everything by d.]

b. [Euclid's Lemma] If $\gcd(a, b) = 1$ and $a|bc$, then $a|c$. [*Suggestion.* Apply Bézout's Lemma to $\gcd(a, b)$, then multiply everything by c.]

c. Recall that a positive integer $p \geq 2$ is called *prime* if its only divisors in $\mathbb{N}$ are 1 and p. Use part **b** to show that if p is prime and $p|ab$, then $p|a$ or $p|b$.

Exercise 2. Use the fact that 1 has order 30 to compute the order of every element in $\mathbb{Z}_{30}$.

Exercise 3. Lang, Exercise II.1.16.