Modern Algebra
Spring 2023

Assignment 8.2
Due March 8

Exercise 1. Let $f : G \to H$ be a group homomorphism. Use the First Isomorphism Theorem to show that

$$[G : \ker f] = |\operatorname{im} f|.$$

Exercise 2. Let A be an additive abelian group. Given $n \in \mathbb{N}$, recall that

$$A[n] = \{a \in A \mid na = 0\}$$

is the *n-torsion subgroup* of A. Use the endomorphism $A \to A$ given by $a \mapsto na$ and the First Isomorphism Theorem to show that $A/A[n]$ is isomorphic to a subgroup of A.

Exercise 3. If G is a group and $a, b \in G$, the *commutator* of a and b is

$$[a, b] = aba^{-1}b^{-1}.$$

The *commutator subgroup* of G is the subgroup generated by the set of commutators in G:

$$[G, G] = \langle\, [a, b] \mid a, b \in G\rangle.$$

Use the First Isomorphism Theorem to show that if $f : G \to H$ is a homomorphism and H is abelian, then $[G, G] < \ker f$.