LINEAR ALGEBRA
SPRING 2024

ASSIGNMENT 3.2
DUE JANUARY 31

Exercise 1. Textbook exercise 1.3.14

Exercise 2. Textbook exercise 1.3.23

Exercise 3. Suppose that a 4×6 matrix $A = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5\ \mathbf{a}_6)$ has reduced row echelon form

$$\begin{pmatrix} 1 & 0 & -1 & 1 & 0 & 4 \\ 0 & 1 & 2 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Find a set $\mathcal{B}$ of independent columns of A that is as large as possible, determine the rank of A, and express the remaining columns of A as linear combinations of those in $\mathcal{B}$.

Exercise 4. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 & 5 \\ 2 & 4 & 4 & -2 \\ 0 & 0 & 5 & -10 \end{pmatrix}.$$

a. Find a set $\mathcal{B}$ of independent columns of A that is as large as possible, determine the rank of A, and express the remaining columns of A as linear combinations of those in $\mathcal{B}$.

b. Let B be the matrix obtained from A by reversing the order of its columns. Explain why $C(A) = C(B)$.

c. Repeat part **a** for B. Do you get the same results?