MODERN ALGEBRA
SPRING 2025

ASSIGNMENT 10.1
DUE APRIL 9

Exercise 1. Let G be a finite group of order n. Given $a \in G$, let $\lambda_a \in \mathrm{Perm}(G) \cong S_n$ denote left translation by a. If a has order m, prove that $\epsilon(\lambda_a) = (-1)^{n(m+1)/m}$. [*Suggestion:* Show that the cycles of λ_a are the right cosets of $\langle a \rangle$ in G.]

Exercise 2. Let G be a group. If N, H are subgroups of G, and N is normal in G, prove that $N \cap H$ is normal in H. Hence, if we also assume that $N < H$, we conclude that $N \triangleleft H$.

Exercise 3. Prove that $H = \{(1), (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$ is a normal subgroup of S_4. Since $H < A_4$, this proves that $H \triangleleft A_4$ (by the preceding exercise), and hence that A_4 is *not* simple. Prove that $S_4/H \cong S_3$ and $A_4/H \cong \mathbb{Z}_3$.