Exercise 1. Let p be an odd prime. Implement the proof of Lemma 2 (Lemma II.7.3 in Lang) for the abelian p-group $A = (\mathbb{Z}/p\mathbb{Z}) \oplus (\mathbb{Z}/p^2\mathbb{Z}) \oplus (\mathbb{Z}/p^3\mathbb{Z})$, taking $b = (1, 2, 1)$.

Exercise 2. Let p be an odd prime. Show that the *Heisenberg group* (over $\mathbb{Z}/p\mathbb{Z}$),

$$H(p) = \left\{ \begin{pmatrix} 1 & a & b \\ & 1 & c \\ & & 1 \end{pmatrix} \middle| \, a, b, c \in \mathbb{Z}/p\mathbb{Z} \right\},$$

is a non-abelian p-group of order p^3. Show that every nonidentity element of $H(p)$ has order p.